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DYNAMICS OF SHEARING-RELAXATION OSCILLATIONS AT THE PLASTIC FLOW OF CRYSTALLINE SOLIDS

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Розглядається вплив низькочастотної вібрації на процес пластичної деформації твердих кристалічних тіл. Аналітичним шляхом дана оцінка умов, при яких внаслідок перерозподілу у часі навантаження, діючого на об'єм, що деформується, відбувається резонансне підсилення інтенсивності зсувно-релаксаційних коливань пластичної течії. Це дозволило обґрунтувати доцільність використання низькочастотної вібрації певної якості для стимулювання процесу пластичної деформації конструкційних матеріалів. Одержані результаті можуть знайти практичне застосування при виробництві виробів точного машинобудування, приладобудування, нафтогазової та аерокосмічної промисловості, енергетики, а також у галузі нанотехнологій.

Ключові слова: *хвиля пластичної деформації, зсувно-релаксаційні коливання, функціональний відгук, резонанс.*

It is considered the influence of a low-frequency vibration on the process of plastic deformation of solid crystalline bodies. Analytic estimation is given to the conditions, which further resonant amplification of shearing-oscillation intensity of plastic flow owing to reallocation in time of load affecting the deformable volume. This allowed grounding practicability of applying the low-frequency vibration of certain quality to stimulate the process of plastic deformation of structural materials. The results obtained can be used while producing items for precision engineering, in tool engineering, oil-and-gas and aerospace industries, power engineering as well as in nanotechnologies.

Keywords: *plastic flow, shearing-relaxation oscillations, self-organization, plastic deformation wave, functional response, resonance.*

The research by W. Lüders, G. I. Tailor, A. F. Ioffe and other scientists have shown that plastic flow of crystalline solids occurs due to the shears of some elements of the deformed volume structure against the others along distinct crystallographic slip planes. The appearance of the shears is related to complying with the yield condition by Huber – Mises – Hencky and being subject to the law by Schmid – Boas. As the research by A. F. Ioffe and P. Ehrenfest showed, shear deformation develops periodically and spasmodically. At the macrolevel, it results in the phenomena of interrupted yield (Porteven – Le Chatelier effect). This peculiarity of the plastic flow was first explained on the basis of the Frenkel – Kontorova atomic model of deformation behaviour of solids; later it was linked to the dislocation motion. Its detailed study allowed E. McReynolds to come to the conclusion of the wave nature of plastic flow. The truth of the conclusion was proved by the results of the experiments conducted by J. F. Bell, O. W. Dillon, M. J. Kenig and others [1].

According to the contemporary idea, the shear deformation is the result of a series of consequent acts of relaxation of internal stresses which occur in the solid objected to the loading beyond the yield point. During these acts original shears appear in the fields near the stress concentrators. Each of them induces the appearance of the following one. Initially, the shears are chaotic. However, as the deformation develops, they correlate with each other causing avalanche-like development of multiplex sliding. As a result of such self-organization there is formed a macroscopic wave of shearing deformation (the Lüders – McReynolds wave). It cannot be identified with dispensing along the plasticity front of the sample. It is a complicated spatio-temporal structure that reflects the synergetics of interaction of the whole totality of hierarchical structural levels of a solid and includes a mechanical motion of a substance, processes of relaxation and dissipation, strain hardening etc. The Lüders – McReynolds's shearing-relaxation wave length linearly depends on the size of crystalline grains and its phase speed is estimated within 10^{-3} to 10^{-2} *m*/*sec* [2].

As a rule, the plastic deformation of the solid is considered via the models of ideal or hardening rigid-plastic medium. However, according to W. Johnson, they do not reflect the course of plastic flow sufficiently [3]. For example, these concepts suggest that the components of a stress deviator are directly proportional to the constituents of a strain deviator. It follows that the immediate value of characteristics of a stressed state of a crystalline solid are determined by its degree of deformation. This approach allows describing shearing character of plastic flow, but it does not consider the relaxation mechanism of the process which causes exponential rather than directly proportional dependence between the values stated. While conducting macroscopic studies of the simplest cases of loading, one can neglect such an error. This enables the application of the hypothesis of uniform stress-strain curve (hardening curve) which is described with the homogeneous scalar function. The data obtained this way can be applied to more complicated loadings [4]. Meanwhile, J. F. Bell's research showed that due to shearing-relaxation effects, the hardening curve is actually quantized, i.e. its characteristics change discretely [1]. The difference between the experiments results and the theory provides the evidence of the facts that the hypothesis of the uniform stress-strain curve and model concepts linked to it is rather limited. Their use simplifies physical equations of interaction considerably; however, it does not always allow determining the precise values of stress-strain state characteristics. The example of alternating-sign or repeated deformation is particularly evident. The latter case gains a significant value given the plastic flow itself is a periodic process.

Periodicity and natural ability of plastic flow to internal self-organization can be shown with a mechanic model which consists of rheological bodies of Hooke, Kelvin – Voigt and Saint-Venant connected consequently. This dissipative self-oscillatory system is a logical development of a model of the standard viscoelastic body [5]. Under the influence of an external force its first two elements show real elastic, viscoelastic and relaxation processes, and the last one shows the mechanical motion of a substance which appears after exceeding the yield point of the working stresses. The model suggested also takes the shearing-relaxation character of the plastic flow and its periodicity into consideration. This enables its application for the analytical description of this process regarding its wave nature. Let us demonstrate this possibility.

Applying the general rules of the rheological modelling we will get the following: the extreme cycle, to which the shearing-relaxation oscillations of the system suggested endeavour asymptotically, has its natural frequency

$$\nu_0 = 1/\tau = G/\eta \text{, Hz} \quad (1)$$

where τ is the time (period) of stress relaxation, *sec.*; *G* is the shear modulus, *Pa*; η is the dynamic viscosity coefficient (the coefficient of internal friction), *Pa · sec*.

The numerical values of these quantities for some materials that are at room temperature are given in the Table 1 [6].

Table 1. The physical characteristics of metals

Material	Yield point σ_s, *megPa*	Shear modulus *G, GPa*	Dynamic viscosity η, *GPa · sec*	Relaxation time τ, *sec*	Shear frequency ν_0, *Hz*
Aluminum	20,00	26,00	1,04	0,04	25,00
Deformed copper	264,63	48,00	4,83	0,10	9,79
Brass	220,00	39,07	2,42	0,06	16,14
Nickel	340,00	94,80	6,30	0,07	15,05
Steel 1008	176,50	67,60	8,11	0,12	8,33
Steel 302	196,10	77,00	10,78	0,14	7,14
Steel 1045	353,00	78,00	7,80	0,10	10,00

According to the formula (1) it follows that quantity ν_0 is strictly constant under stationary physical conditions, but ascends with the load increased. This is connected with hardening.

In the shear process, working stresses quantity σ changes within the yield point (here it is suggested that quantity σ is the operator of stress tensor σ_{ij}). At the same time at every moment of t the following equality holds

$$H\vec{f} = \vec{F} \text{, Pa} \quad (2)$$

where the left part is a vector field of internal stresses whose value is determined by matrix H, and $\vec{F}$ is the vector function of external force action expressed via stresses [7].

Proceeding to the scalar form, we get the following from the analysis of the deformation behaviour of the rheological model suggested:

$$\ddot{\sigma} + 2\beta\dot{\sigma} + \omega_0^2\sigma = F(\sigma;\ \dot{\sigma}) \text{, Pa/sec}^2 \quad (3)$$

where β is the decay coefficient of shearing-relaxation acts whose value is defined by viscous properties of the environment, sec^{-1}; $\omega_0 = 2\pi\nu_0$ is the natural cyclic frequency of the shearing-relaxation oscillations, $radn/sec$; $F(\sigma;\dot{\sigma})$ is the function of external forcing action, written in an impulse form.

As a matter of fact, equation (3) is an expression of Newton's second law for the periodical motion of the deformed elastoviscoplastic medium.

Let us approximate the relation $\sigma(t)$ with a harmonic function. On the one hand, such a possibility directly results from the deformation behaviour analysis of the Frenkel – Kontorova model adapted for the macrolevel of the structure of substance on the basis of the classical visco-plasticity theory of Thurston – Maxwell – Mises [1]. On the other hand, one can arrive at the same idea based on the contemporary wave theory of plastic deformation. As a result, a particular solution of equation (3) will have the form:

$$\sigma = \sigma(t) = \sigma_a \sin(\omega t - \varphi_0), \text{ Pa} \quad (4)$$

where σ_a is an amplitude of working stresses; $\omega = (\omega_0^2 - \beta_0^2)^{0,5}$ is cyclic frequency of steady shearing-relaxation, rad/sec; $\varphi_0 = arctg\,\frac{2\omega\beta}{\omega_0^2 - \omega^2}$ is a phase displacement of the size of working stress alterations σ, which appear due to the retardation processes, rad.

With the attenuation of shearing-relaxation oscillations quantity size σ_a is decreased. According to the Oding – Nadai's equation [8]

$$\sigma_a = \sigma_a(t) = \sigma_0 \exp(-t/\tau) = \sigma_0 \exp(-\omega t/2\pi),$$

where σ_0 is the initial biggest value of amplitude of working stresses σ; and time t changes within $0 \le t \le \tau$.

Expanding the function of external influence $F(\sigma; \dot{\sigma})$ into Fourier series, we get

$$\sigma = \sigma(t) = \frac{X(\sigma_a)\sin\omega t - Y(\sigma_a)\cos\omega t}{2\beta\omega} + Z(\sigma; t), \quad (5)$$

where $X(\sigma_a) = \frac{\omega}{\pi}\int_0^{2\pi/\omega}[F(\sigma; \dot{\sigma})\cos\omega t]dt$ and $Y(\sigma_a) = \frac{\omega}{\pi}\int_0^{2\pi/\omega}[F(\sigma; \dot{\sigma})\sin\omega t]dt$ are spectral members of harmonic analysis of function $F(\sigma; \dot{\sigma})$, while $Z(\sigma; \dot{\sigma})$ is the difference between the exact solution of equation (3) and approximating oscillation (4).

Due to the strain hardening phase oscillations $\varphi(t) = \omega t - \varphi_0$ changes. To determine it, we can use, for example, Krilov – Bogoliubov method. As a result, we get

$$\dot{\varphi} = d\varphi/dt = \omega + [X(\sigma_0)/\sigma_0\omega].$$

Quantity σ is a multicomponent function [7]. Apart from the time, it depends on the physical properties of the materials deformed, the degree of its deformation, external force influence characteristics, and temperature. At every moment of time the totality of values of these parameters determines the momentary functional response of substance at every point of the deformable volume. In turn, the totality of momentary functional responses of all its points forms mathematical space of the integral response of the system to the external influence [2]. The obtained solution of equation (3) implicitly considers all the parameters, except the deformation degree. The lack of the deformational element does not allow finding out connections between quasi-harmonic changes of acting stresses and natural oscillations of plastic flow. To solve this task, let us apply the mathematical method of perturbation according to Poincaré – Liapunov's interpretation and analyse the function of alteration rate of working stresses $\dot{\sigma}$.

At the constant temperature

$$\dot{\sigma} = d\sigma/dt = f(\sigma; \varepsilon; t), \text{ Pa/sec} \quad (6)$$

where ε is an operator of small deformation tensor ε_{ij}:

$$\varepsilon = \varepsilon_{ij}(t) = 0,5\left[\frac{\partial u_i(t)}{\partial x_j} + \frac{\partial u_j(t)}{\partial x_i}\right], \quad (7)$$

which describes change of the current or initial coordinates x_{ij} of material points of the deformable volume with their displacements u_{ij} during the deformation process.

Equation (6) is equivalent to the system of equations. Let each of its solutions valid for every ε_{ij}, cohere with certain partial solution (5) of the equation (3). With incremental strain by the value $\varepsilon(t_0) + \delta\varepsilon(t)$, stress alterations will make $\sigma(t_0) + \delta\sigma(t)$. Then the system (6) in perturbations (variations) will be as follows:

$$\delta\dot{\sigma} = \delta\dot{\sigma}(t) = f[t;\ \sigma(t_0) + \delta\sigma(t);\ \varepsilon(t_0) + \delta\varepsilon(t)] - f[0;\ \sigma(0);\ \varepsilon(0)], \text{ where } \delta\sigma(0) = 0. \quad (8)$$

These equations are exact. However, if matrix function (6) is differentiable, while factoring the right side of the equations (8) into Taylor series we can confine ourselves to the members of the first order. As a result we will get the linear system

$$\delta\dot{\sigma} = \dot{\Psi} = \delta\sigma\frac{\partial f}{\partial \sigma} + \delta\varepsilon\frac{\partial f}{\partial \varepsilon}, \text{ Pa/sec} \quad (9)$$

where $\Psi = \Psi(\delta\sigma;\ \delta\varepsilon) \sim \exp(-\omega_0 t/2\pi)$ the matrix of its nontrivial solutions, exponentially tending to zero with $t \to \tau$, *Pa*.

With $\delta\sigma_{ij} << |\sigma_{ij}|$, we can ignore the error of this substituting. After being integrated, the expression (9) amounts to the Oding – Nadai equation for deformed state of crystalline solid. The system (9) allows defining the degree of approximation (the first order perturbation) for the quantity $\delta\sigma_{ij}$; however, in this case it is necessary to know $\delta\varepsilon_{ij}$.

The analysis of the rheological model suggested proves that amplitude of steady-state shearing-relaxation oscillations is [2]

$$l_S = l_S(t) = \frac{\sigma_S S}{\Delta LG}\left[1 - \exp\left(-\frac{\omega t}{2\pi}\right)\right], \text{ m}$$

where S is the cross-section of a deformable body m^2; ΔL is the absolute degree of its deformation, *m*; $\Delta L\Delta G$ is reduced stiffness of the material *N/m*.

Then incremental strain in the course of a separate shear is

$$\delta\varepsilon = \delta\varepsilon_{ij}(t) = 0{,}5\left(\frac{\partial}{\partial x_j}\left[u_i(0) + \delta u_i(t)\right] + \frac{\partial}{\partial x_i}\left[u_j(0) + \delta u_j(t)\right]\right), \quad (10)$$

where $0 \le t \le \tau$; $0 \le \delta u_{ij}(t) \le l_S$.

Considering the tensor (10) the differential equations (8) and (9) allow to pass from the narrow solution of equation (3) relatively working stresses to matrix functions, which include deformation components.

The results of the presented physical and mathematical analysis allow evaluating the character of the external deforming influence on the dynamics of natural shearing-relaxation oscillations of plastic flow. As the research has shown, the influence is the most significant during the vibratory loading [6].

Let us assume that the crystalline solid be influenced by deformation effect with frequency ω_a. It causes stresses in the volume of the substance whose volume changes with amplitude σ_0 periodically. The deformation that occurs can be presented as the result of interaction of two oscillatory systems with different power: the more powerful one is a deforming device and the less powerful one is a self-oscillatory system of deformable material. According to the theory of harmonic oscillators, during the interaction the more powerful system seizes the oscillations of the less powerful system and adjusts them to its own frequency [9]. In actual practice, the possibility and final outcome of such self-organization are influenced by the condition of the deformable material at starting point of the perturbation as well as by running values of stresses σ, developed by the external action and the loading frequency ω_a. Depending on this, the functional response of the physical system of a deformable material can be periodic, quasi-periodic or chaotic.

Let us denote resistance of solid crystalline substance to plastic deformation with σ_r. According to Newton's law of internal friction

$$\sigma_r = \eta\nabla\upsilon = \eta/\tau = \eta\nu_0 = \eta\dot{\varepsilon}, \text{ Pa} \quad (11)$$

where $\nabla\upsilon$ is the velocity gradient which appear between the discrete points of deforming volume during the plastic flow, sec^{-1}.

If $\sigma_0 >> \sigma_r$, the deformation processes in the working material are captured by external vibratory influence regardless of value of ω_a (here, it is presumed that due to the yield drop $\sigma_r \ne \sigma_S$). From the solution of the differential equation (3) it follows that herewith there appears a periodical functional response of the system with frequency ω which is multiple of frequency ω_a. As a result, the plastic flow loses its initial self-oscillating character and is completely subordinate to the external constraining influence. This is attended with active hardening which gives evidence of inefficient use of natural plastic properties of working material; it is also attended with increase in degree of deformation inhomogeneity and formation of considerable residual stresses.

If $\sigma_0 \sim \sigma_r$, then the external forcing is too weak to overcome the natural tendency of the deformation processes to develop spontaneously with their free-running frequency ω_0. As a result, a two-component quasi-periodic functional response with basic frequencies ω_a и ω_0 appears. In its spectrum there will certainly be components for which

$$\omega_a = n\omega_0 + \beta \text{, } sec^{-1}\text{, where } n = 1, 2, 3... \quad (12)$$

When the condition (12) external vibration influence captures the natural oscillations of plastic flow and synchronizes them relatively to own frequency ω_a. The width of the frequency band of synchronization is determined by the properties of the deformable medium and does not depend on the way of forming.

For convenience of the further reasoning, let us present the function of external influence in the equation (3) as $F(n\omega_0 t + \alpha;\ \sigma_0;\ \varepsilon)$. Then, from equation (6) we get the rate of cyclic changes of the absolute value of working stresses

$$\dot{\sigma} = f(t;\sigma) + F(n\omega_0 t + \alpha;\ \sigma_0;\ \varepsilon)\text{, } Pa/sec \quad (13)$$

where α is the initial phase of the external force impact, *radn*.

The changes taken allow applying Laud's algorithm to the perturbation method which has already been used here [7]. With it, we find that the first order perturbation (the first variation) for the quantity σ will have the form:

$$\delta\sigma(t;\sigma;\varepsilon) = C + \dot{\Psi}\int_0^{2\pi/\omega_0} \dot{\Psi}^{-1}\left[F(n\omega_0 + \alpha;\ \sigma_0;\ \varepsilon) - \frac{\beta\dot{\sigma}(\tau)}{n\omega_0}\right]dt\text{, } P \quad (14)$$

where C is the matrix of initial conditions with components $\sigma_{ij}(t_0)$ and $\varepsilon_{ij}(t_0)$, *Pa*; $\sigma(\tau)$ is the stress, which, according to the Oding - Nadai theory, is saved in deformable volume of solid crystalline body after the completion of the first stage of a single relaxation act.

According to the Laud algorithm function (14) will be periodic if all the components of the matrix C, excluding the first one, equal to zero, while α and β are such that

$$\int_0^{2\pi/\omega_0} \psi\left[F(n\omega_0 t + \alpha;\ \sigma_0;\ \varepsilon) - \frac{\beta\dot{\sigma}(\tau)}{n\omega_0}\right]dt = 0. \quad (15)$$

Quantity ψ is an operator of the upper row of the conjugate matrix Ψ^{-1} of nontrivial solutions of the equation system (9). It expresses the periodic function that is changed with frequency ω_0 at $\sigma_0 \sim \sigma_r$. According to the Floquet's theorem $\psi\sigma(\tau) = 1$. Then, if the perturbation frequency ω_a is too small or large in comparison with the decay coefficient β, any initial phase of external influence α doesn't satisfy to the condition (15). In other words, the perturbation frequency goes beyond (12) of the synchronization band and the capturing of own oscillations of plastic flow by external vibration influence appears impossible. To implementation at least the single capture it is necessary that

$$\min \Omega(\alpha) < \beta < \max \Omega(\alpha), \quad (16)$$

where

$$\Omega(\alpha) = \left(\frac{n\omega_0^2}{2\pi}\right)\int_0^{2\pi/\omega_0} \dot{\psi}\, F(n\omega_0 t + \alpha;\ \sigma_0;\ \varepsilon)dt\text{, } sec^{-1}.$$

The general condition (16) is connected with known decay dependence in the medium on the loading frequency [7]. Its substitution into formula (12) shows that with small perturbations when $\sigma_0 \sim \sigma_r$, the frequency which allows the external vibrating influence to capture external deformation processes is within the limits of

$$n\omega_0 + \min\Omega(\alpha) < \omega < n\omega_0 + \max\Omega(\alpha). \quad (17)$$

Inequality (17) characterizes the frequency bandwidth of synchronization, within which condition (12) is met. If $\omega_a \in \Delta\omega$, the development of each following shear starts when the previous act of the macroscopic deformation is only coming to its end. Thus, the external forcing appears to correlate with the natural course of the macroscopic shearing-relaxation processes in a deformable volume of substance; as a result, between there appears certain resonance. With the resonance, oscillations with resonant frequency are formed in the substance.

$$\omega_{res} = (\omega_0^2 - 2\beta^2)^{0,5} < \omega_0.$$

As it can be seen from formula (11), in this case the resistance of crystalline substance to plastic deformation is decreased. The decrease results in functional response reinforcement (14). Physically, this is expressed in increasing intensity of shears. One can easily come to this conclusion analysing the well-known formula for determining the degree of material deformation [7]:

$$\varepsilon = \int_0^t \xi(t)dt,$$

where the intensity of deformation rates $\xi = 2[|I_2(D)|/3]^{0,5}$; here $I_2(D)$ is the second invariant of the deformation rate deviator, sec^{-1}, t is the time, c.

The studies have shown that, for example, in case of resonant vibro-deformation of metals and their alloys the degree of decrease of deformation resistance value makes 20 to 50%. The discovered phenomenon was named the resonant vibro-plastic effect [6]. Mathematical expression of conditions of its occurrence is inequality (17). It is obvious that this effect is the most pronounced with the main resonance when $n = (\omega_a - \beta)/\omega_0 = 1$. According to the formula (11), in this case the deformation resistance σ_r possesses the least value possible under the given conditions. With the growth of resonance multiplicity n its effectiveness will gradually decrease. This fact can be explained by the following. As n increases, the degree of mismatch between external forcing and natural course of the shearing-relaxation oscillations in the deformable volume of substance grows as well. Simultaneously, frameworks of the synchronization band reduce. Indeed, equation (12) implies that $\Delta\omega = \mathrm{sign}[(\omega_a - \beta)/2n]$. The interaction between the stated factors conditions the limitation of the ability of the two oscillatory systems to synchronize spontaneously.

If $\omega_a \sim \omega_0$, but $\omega_a \notin \Delta\omega$, the shearing processes take form of beats. With them, the amplitude of shears l_s changes periodically. In this case, the plastic flow obeys the laws of motion of the medium in the oscillating field. As P. L. Kapitsa showed, the material points of such medium follows a certain complicated main trajectory and simultaneously makes oscillations against it. Considering the whole deformable volume, it is worth concluding that occurrence of beats gives evidence of increasing entropy. This complicates the natural self-organization of shearing deformation and impedes formation of a steady macroscopic wave of plastic flow. Now, the shears become more chaotic.

According to Newton's third law, growing chaotic character of shifts leads to increasing deformation resistance σ_r. At the same time there occurs inhibition of the translational component of the mechanical motion of substance and triggering development of compensatory orientation turns of the crystalline structure elements in the deformable volume against the axis of strain. In the stress-strain diagram, these changes are expressed in acceleration of transition from the easy slip stage to the linear and then parabolic hardening stage.

Fast growth of the deformation resistance occurs when natural oscillations of plastic flow appear in the antiphase considering the external forcing. The theory of harmonic oscillators shows that herewith the resistance of the synchronized oscillatory system to the external forcing can be higher than under the fixed load [6]. As applied to the plastic vibro-deformation of crystalline solids, this means that there are certain technological regimes under which vibration does not only contribute to the plastic flow of pre-set material, but also obstructs it, decelerating Lüders – McReynolds wave front propagation [6].

With intermediate values σ_0 the functional response will be neither periodic nor quasi-periodic. In this case in matrix C of initial conditions for function (14) there appear additional nonzero components. Essentially, they are random quantities for real crystalline solids. According to Laud's algorithm, their occurrence leads to non-observance of condition (15) of perturbation periodicity (14) and therefore in the matrix Ψ of solutions of the equation system (13) there appear anharmonic members. At the same time, uncertainty enters the capture condition (12). As a result, in the spectrum of functional response there can be observed superimposition of separate synchronization bands and their imposition on the basic two-component quasi-periodic response (noise) while the response itself becomes chaotic.

The chaotic response occurrence prevents the correlation of external and internal processes of deformation. This eliminates a possibility of resonant amplification of any of the harmonics (12) in the perturbation spectrum (14). Moreover, it is accompanied by increasing velocity gradient $\nabla\upsilon$ of the deformable volume points (the lower the degree of correlation is, the larger is the gradient). formula (11) shows that its increase, in turn, conditions the growth of deformation resistance σ_r.

It is obvious that the biggest mismatch occurs under the fixed load. According to thermodynamics, such a method of triggering the plastic flow results in energetic supersaturation of the deformable volume of a crystalline substance [9]. At first glance, this should contribute to increasing intensity of plastic flow. At the same time, according to the law of conservation of energy and principle of entropy increase, high energy content conditions a sharp increase of its entropy. In the limit, it results in a complete loss of the plastic flow and lack of its initial self-oscillatory character. Under these conditions, as well as under $\sigma_0 >> \sigma_r$ the internal processes of self-organization of the plastic flow are also completely suppressed by the external forcing.

Another example of a complete loss of capability for self-regulation by the plastic flow is the influence of strong live loads on a solid, for example, on impact. In this case, similar to beats, a blockage of a translational component of deformation occurs as well as its compensation due to the development of a rotary component. The difference between them is that on impact because of high rate of loading the translational component interlocks by a factor of several times faster. As a result, almost immediately after an impact action, shearing deformation in the volume of the substance develops, mainly due to the rotary component of its motion. Simultaneously, a vortex dissipative structure appears which is responsible for the parabolic stage of the hardening curve. Mathematically, it is described by equations [2]

$$\frac{\partial}{\partial t}\left[\frac{1}{l^{2D_f}}\Phi^{ij}\lambda_i^\alpha\left(D_\mu\lambda_j^\alpha\right)\right]\cong \text{grad}\,\Phi^{ij}\lambda_j^\alpha\dot{\lambda}_j^\alpha\,; \quad (18)$$

$$\text{rot}\,\frac{1}{l^{2D_f}}\Phi^{ij}\lambda_i^\alpha\left(D_\mu\lambda_j^\alpha\right)\neq 0\,, \quad (19)$$

where l is a linear dimension of mesoscopic element of deformable volume of substance[1]; D_f is its fractal dimensionality; Φ^{ij} is a diffusion flux of lattice defects, which appears in the deformable volume of solid at its plastic flow; λ_i^α and $\lambda_j^\alpha=\lambda_i^\alpha\Phi^{ij}$ data of a mesoscopic component of the substance volume; D_μ is covariant derivative.

The left part of equation (18) characterizes the rate of a diffusion flux of defects Φ^{ij}, while its right part characterizes the gradient of their source in the head of Lüders bands, also known as slip bands which acts as a stress concentrator. Non-vanishing rotor of defect flux (19) means that the wave of plastic deformation moves along the sample with heavy attenuation, i.e. according to J. B. Friedman's theory, it is unstable.

The described regularities of deformation behaviour of crystalline solids occur under the plastic deformation. That is why they cannot be regarded as a result of, for example, Bauschinger effect. Another peculiarity is that they are available in a low-frequency (infrasonic) region. The effect of amplification of intensity of plastic flow by initiating internal resonances in the deformable volume of a solid significantly differentiates resonant vibro-deformation from other known methods of vibration impact on materials. The difference is that traditional methods are aimed at amplification of individual impulses of external influence by initiating resonant processes in the system of a function element of a deforming device or by adding direct (active) and reflected (reactive) elastic waves which appear in the sample body at loading. Meanwhile, under vibro-plastic resonance the external influence activates the natural deformation processes inside the processed material which allows it to exhibit its natural capability of shape-forming more amply. This confirms M. Ashby and R. Verall's idea stated in 1973 that in fact crystalline solids possess a much bigger reserve of plasticity which mankind has not learnt to use efficiently yet [10].

The mechanism of vibro-plastic resonance appearance has a physical nature completely different from the processes which accompany cyclic softening. Indeed, under cyclic softening destruction of dislocation clusters occurs; the clusters are formed in the process of deformation and block further work of dislocation sources with their force fields. By contrast, matching of external forcing frequency with natural frequency of periodic elementary shears in plastically deformable material allows separate, initially chaotic, primary shifts to correlate between each other and form wave front of coordinated plastic flow. Due to such self-organization the rate of formation in the deformable volume of the dislocation blocking layer decreases. Correspondingly, counteraction degree from force fields is reduced as well. In outward appearance, this is expressed in a decreasing degree of strain hardening per one cycle of loading. Under subsequent loadings, the counteraction of dislocation clusters is overcome at the expense of resonant amplification of shift intensity.

The excitation of vibro-plastic resonances does not exclude a possibility of simultaneous ultrasonic stimulation of plastic flow. The ultrasonic radiation influences a solid at an atomic-molecular level. The absorption of it by a substance causes the increase of potential energy of both individual atoms in the crystal lattice, and their various mutual configurations including dislocations. At a macroscopic level it is expressed in increasing internal energy of the total deformable volume. Thus, in its physical substance, ultrasonic stimulation of the plastic flow is equal to the process of "conventional" heating of workpiece, however, being different from the latter one having larger technical complexity. Like thermal influence, it causes reduced activation energy necessary for rupturing internal bonds in the deformable volume of substance and for starting its mechanical motion. As a result, according to Boltzmann's principle of time-temperature superposition, material appears to be prepared to perceive a vibration stimulus [9].

Maintaining the natural course of the shift-relaxation oscillations in the volume of crystalline solid by providing the resonant vibration stimulus furthers the increase of a degree of deformation homogeneity of workpieces. This allows applying the method of resonant vibro-deformation while producing items of precision engineering, for example, for the manufacture of high-speed shafts of aviation fuel pumps, plungers of axial-piston hydraulic machines of high pressure, precision shafts and spindles of instrumental stocks on supports with gas lubrication, very precise guides of mobile elements of various executive mechanisms and others. Thereby, the method of vibro-deformation can be considered technology which corresponds to modern trends aimed at replacing traditional technologies of cutting structural materials with more efficient working by pressure. It can be accomplished in the process of stamping, pressing, draw-forming, drawing, extrusion and others.

[1] Mesoscopic is the volume element that includes cells of Frank - Reed dislocation network, deformation domains, sub- and crystal grains, their conglomerates, etc.

The analysis suggested is specified as applied to the processes which occur at the macrolevel of structure of crystalline solid and is by no means athermic. It implies introducing corresponding adjustments in view of temperature changes of physical characteristics of the worked material.

CONCLUSIONS

1. Plastic flow of crystalline solids has a wave nature. It is a synergetic process of the evolution of shear stability loss which gradually develops at micro-, meso- and macroscopic structural levels of a body.

2. Mechanical motion of substance under the plastic flow has oscillating character. In the base of every oscillation there are periodic elementary acts of stress relaxation. Their frequency is determined by the properties of deformable material. It remains stable under permanent physical conditions.

3. Oscillations of plastic flow is a series of elementary shifts of one elements of the structure of the deformable volume relatively to the others. If their development is turned out difficult, the mechanical motion of matter occurs at expense of orientational crystallographic turns of the same elements with respect to the deformation axis. Together these two components form a translationally-rotating wave of plastic deformation (the Lüders – McReynolds wave).

4. Under vibration stimulus with frequency which is resonant to its natural frequency of plastic flow oscillations in deformable volume, resonance appears. Its stimulation furthers maintenance of natural course of deformation processes in the crystalline solid. Due to this, the deformable material gets an opportunity to exhibit its natural plastic properties more amply.

5. Vibro-plastic resonance occurs in the frequency interval (within limits of frequency band of synchronization), whose width is determined by the properties of the deformable material and does not depend on a deformation method. It can be observed under conditions of both main and multiple resonances; their efficiency decreases as phase difference between two oscillatory processes increases.

6. Overrunning limits of frequency band of synchronization is accompanied by mismatch of external forcing and natural oscillations of plastic flow. This impedes the natural development of the processes of self-organization of the deformable volume structure and results in increased resistance of the material to plastic deformation.

Excitation effect in the volume of crystalline solid of vibro-plastic resonance is related to the decreased value of deforming force and a increased degree of homogeneity of deformation. This gives ground to expect that the method suggested can be applied while producing items for precision engineering, in tool engineering, oil and gas and aerospace industries, energetics, as well as in nanotechnologies.

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